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REVIEWS

Solitons. By P. G. DRAZIN. London Mathematical Society Lecture Note Series no. 85, Cambridge University Press, 1983. 136 pp. £7.95 (pbk).

In the last decades two trends of a universal nature have emerged from computational studies of nonlinear dynamical problems:

(i) the solutions to the problems may tend towards organized states, or, oppositely,

(ii) the systems may respond in a disordered manner to certain stimuli (chaos, blow-up etc.).

An idealized ordered state is the soliton wave discovered and named in 1965 by Martin Kruskal and Norman Zabusky. Solitons are solitary waves – i.e. localized waves of permanent form – with the additional property that the waves may interact without losing their shape as if the superposition principle were valid for this non-linear phenomenon. In applications the *exact* solitonic property is of course never encountered, just as the exact plane wave, say, does not occur in linear field-theory applications. Nevertheless, both the soliton and the plane wave are fundamental concepts in nonlinear and linear field theory. Recent literature in physical, mathematical, engineering, chemical and biological disciplines shows an exponentially increasing number of applications of solitons. Waves that only possess a qualitative solitonic property are also important in applications, and such waves are often also denoted solitons in a looser sense of the word.

The present lecture notes deal with the mathematical treatment of exact solitons. The major part of the book is devoted to the inverse-scattering method invented by Kruskal and his associates in 1967 for the Korteweg–de Vries equation. The method has later been generalized to a host of other nonlinear partial differential equations and systems of such equations, and is now considered a major achievement of 20th century mathematics. The discovery of the inverse-scattering method is a classic example of how computational results may lead to the development of new mathematics, just as observational and experimental results have done since the time of Archimedes.

The notes consist of a preface, eight chapters, an appendix, a subject index and a relatively short bibliography (99 entries), as well as a motion-picture index. Chapter 1 describes John Scott Russell's observations of solitary channel waves in 1834. In his later water-tank experiments he discovered the solitonic effect that an initial disturbance evolves into a number of solitons plus additional linear oscillations. This result was essentially forgotten until Kruskal and Zabusky did their computer study in 1965, inspired by the Fermi–Pasta–Ulam model of phonons in an anharmonic lattice. Chapter 2 deals with travelling wave solutions to the Korteweg–de Vries equation in the form of single waves (solitary waves) and wavetrains (conoidal waves). The existence of infinitely many conservation laws – treated in Chapter 3 – is intimately related to the soliton property. Chapter 4 (the main chapter) gives a thorough introduction to the inverse-scattering method for the Korteweg–de Vries equation. The method can be viewed as a nonlinear generalization of the Fourier-transform method. The final inversion back to real space is achieved through solution of the Gel'fand–Levitan–Marchenko integral equation. This equation is derived in the appendix by means of Balanis' method. The chapter demonstrates the application of the inverse-scattering method for various initial value problems in a number of instructive examples. The notes do not cover problems with periodic boundary conditions. The difference between solitons and shock

waves is illustrated in an exercise on Burgers' equation. Chapter 5 (marked with an asterisk: the symbol is used throughout to indicate a difficult section, paragraph, or problem) gives the underlying operator structure. Lax pairs are given for a number of different nonlinear evolution equations. In Chapter 6 the scene is shifted to the sine–Gordon equation, which has many applications. The application to Josephson tunnel junctions is missing, which is unfortunate because it is particularly in this area that solitonic effects have been demonstrated experimentally. The sine–Gordon equation is solved by means of Lamb's ansatz, the inverse-scattering method and (in Chapter 7) by Bäcklund transformation. Application of an (auto) Bäcklund transformation to a known solution adds a soliton to this solution. The two-soliton solution for the Korteweg–de Vries equation is derived by means of Bianchi's theorem of permutability. Chapter 8 is a very short epilogue on the subject of the notes. A few hot themes from front-line research are touched upon (the application of the Painlevé property in soliton theory for example). Reference to some of the existing state-of-the-art books would have been a useful addition to this section. Each chapter contains a number of illustrative problems. Solutions are not provided to any of them, which is a drawback since many of the problems actually contain results from research papers and therefore may be too difficult for senior students and graduate students. These are among the readers for whom the notes are intended, according to the Preface. Unorthodox problems of the type 'See the animated film..., and relate what you see to the theoretical results you have learnt' are refreshing. Furthermore they reflect the necessity of interacting computational and theoretical work, a new working mode that is characteristic for the field of modern nonlinear dynamics.

The notes are intended for physicists, chemists and engineers as well as mathematicians. This group of researchers, which probably includes most readers of this journal, may now prefer a broader book that also treats systems with almost-solitonic solutions and discusses the competition between ordering and disordering effects.

P. L. CHRISTIANSEN

Structure of Complex Turbulent Shear Flow. Edited by R. DUMAS and L. FULACHIER. Springer, 1983. 444 pp. DM98.00.

This volume contains the proceedings of an IUTAM Conference held in Marseilles, France, from 31 August to 3 September 1982. The book, published by Springer in their familiar conference-proceedings format, has nearly 450 pages and includes 326 figures. The text, line drawings and photographs are reproduced satisfactorily and the minor variations in presentation amongst the authors are not unduly distracting.

The papers are, with only one exception, experimental investigations of complex turbulent shear flows and most describe explicitly, at the organisers' request, the experimental techniques used. The conference organisers selected 37 contributions from the 72 papers submitted and their care has led to the generally high standard of the published papers. The book includes 11 papers on wall flows, 14 on free flows, 6 on supersonic flows and 6 on interacting flows. Summaries and transcripts of audience discussion are presented for the first three groups and for the conference as a whole.

Various wall flows are considered with little overlap amongst the papers. Somewhat traditional papers are presented for turbulent boundary layers on curved, wavy, rotating and compliant surfaces and for separated flows from forward- and backward-facing steps. Longitudinal vortices within turbulent boundary layers are investigated in two more novel papers – in one measuring the vorticity components and in the

other seeking an analogy with the breakdown of a Görtler instability on a concave surface. A further paper addresses the connection between wall pressure fluctuations and velocity fluctuations. There is an interesting, though not extensive, discussion on vortices/vorticity, experimental uncertainty and organised structure in the near-wall region. Professor Launder's request for the measurement of higher-order correlations to assist modellers appears to have been received with little enthusiasm. In fact, little interest is shown in interpreting results using higher-order moment equations in general.

Of the 14 papers on free flows, 3 are on initial instabilities, 3 on the evolution of structures arising from instabilities in various situations, 2 on discerning organised structures in fully turbulent flows and 6 on diverse complex flows including 3-dimensional wakes, and swirling, impinging, wall and confined jets. The discussion section is short but includes two useful, novel contributions from Professor Roshko and Dr Savill both on organised structures in fully turbulent flows.

A well-organised summary, and subsequent discussion, of the 6 papers on supersonic flows points out that the 'dominant preoccupations are: firstly, the mechanisms by which a strong pressure gradient acts on a turbulent flow; secondly, the role of large scale structures on free flows; thirdly the existence of a universal logarithmic law in the case of boundary layers.'

The last 6 papers address boundary layers interacting with another boundary layer, a wake, a jet, freestream turbulence and the effect on a shear flow of external irrotational velocity fluctuations.

The volume ends with a well structured summary and final discussion in which Professor Lumley suggests that many experiments are too complicated and should be simplified and that 'an experiment should always be combined with or illuminate a theoretical construction.' In a similar provocative vein he comments on the lack of innovation in experimental technique and encourages the use of experiments with active and passive scalars, phase changes and chemical reactions as experimental tools. The ensuing discussion is also of considerable interest.

Overall the quality of papers in this volume is high, the subject matter topical and, most importantly, the discussion lively and enlightening. However, the lack of innovation noted by Lumley is too frequently apparent: a surprising observation given the scope and relevance of the topics investigated. Volumes such as this are essential reading for workers in the field though more appropriate for library, and not individual, purchase.

REX BRITTER

SHORTER NOTICES

The Application of Laser Light Scattering to the Study of Biological Motion. Edited by J. C. EARNSHAW and M. W. STEER. Plenum, 1983. 705 pp. $89.50 ($107.40 outside US & Canada)

This volume of NATO Advanced Study Institute proceedings on laser techniques differs from its predecessors by its emphasis on microbiological applications. The biological papers necessarily deal with rather specialized topics, which could be a virtue or a defect, depending on the reader's interests. There are also excellent recapitulations of coherent light-based analysis techniques; some new results are presented and there are copious references. The book contains several articles on micro-fluid-dynamical topics, such as swimming micro-organisms, electrophoretic light scattering, microrheology, thermally excited capillary waves and velocity-profile

analysis in very narrow channels. The organizing thread for this volume's topics is light scattering, but fluid-motion phenomena often provide the setting.

Surface Phenomena in Enhanced Oil Recovery. Edited by D. O. SHAH. Plenum, 1981. 874 pp.

When the natural flow of oil up a well diminishes, secondary methods of recovery must be used. The standard practice is to pump water down suitably located 'injection' wells and thereby to push the oil towards the collecting wells. Such secondary methods leave about two-thirds of the oil in the reservoir, mostly in the form of discrete oil ganglia trapped in the pores of the rock. More sophisticated and more expensive tertiary recovery methods must then be considered. A wide variety of thermal and chemical-flooding processes have been devised and employed, but it is seldom possible to raise the recovery fraction above a half, and there would be enormous commercial rewards for more successful methods. The purpose of chemical flooding is to change the conditions at the oil–water interfaces in the reservoir and to make more mobile the trapped ganglia. The scientific and technological questions involved in chemical flooding were discussed at a symposium at Stockholm in August 1979, and this volume records the two invited surveys and 36 of the contributed papers presented at the meeting. For those concerned with tertiary oil recovery it is an important publication. For those with a general interest in the properties of interfaces some of the papers will provide an introduction to a fascinating area of applied science.

Equilibrium Properties of Fluid Mixtures. 2. By M. J. HIZA, A. J. KIDNEY and R. C. MILLER. Plenum, 1982. 246 pp. $115.

This expensive volume is not a handbook containing data, as one might think from the title, but is a bibliography of publications containing experimental data on thermal and physical properties of multiphase mixtures in equilibrium, with notes and tables indicating the systems and ranges of variables investigated in the various references. The systems referred to are predominantly those of interest in cryogenic applications and in the processing of natural and synthetic fuel gases.

Advances in Two-Phase Flow and Heat Transfer. Edited by S. KAKAC and M. ISHII. Nijhoff, 1983. 920 pp. *Dfl.* 285.00

This is yet another product from the circus of vacation schools summering in Europe. This one appears in the NATO Advanced Science Institute series and it contains the proceedings of an 'Advanced Research Workshop' held at Spitzingsee in September 1982. The material is aimed almost entirely at the nuclear market, with just one or two applications cited of specific practical relevance to the oil and chemical process industries. Thirty-three papers and five workshop panels are offered in two groupings (Volume 1 and Volume 2) with a five page index of authors and subjects to assist the reader. The papers can be classified, roughly, under the headings of: reviews (7); fundamentals (9); phenomenologicals (4); engineering models and tests (9); system codes and design (4). Of these, JFM readers might perhaps be most interested in a few of the fundamental works; in particular: a first-principles approach to modelling of the constitutive equations (Dobran), experiments on the structure of gas–liquid

layered flows (Fabre *et al.*), experiments on large bubbles in a horizontal channel (Sakaguchi *et al.*), and experiments on grid-turbulent bubbly flows (Lance & Bataille).

North Sea Dynamics. Edited by J. SUNDERMAN and W. LENZ. Springer, 1983. 693 pp. £25.10.

This book is a collection of 45 papers given at an interdisciplinary symposium on the North Sea held in Hamburg in 1981. The first paper gives an interesting historical account of the research and describes the development of international collaboration between the countries bordering the North Sea. Despite the interdisciplinary nature of the meeting, most of the papers are about the physics of the sea with 3 of the 4 groupings being currents and water balance, wind waves and storm surges, and transport of momentum, energy and matter. The other section of papers deals with the marine ecosystems and provides an interesting complement to the physical oceanography. The volume is a valuable collection of recent work on the North Sea although it is perhaps regrettable that there are no papers on the chemistry.

Functions of a Complex Variable. Theory and Technique. By G. F. CARRIER, M. Krook and C. E. PEARSON. Hod Books, 1983. 439 pp. $34.95.

This useful applications-oriented textbook on complex variables, which was first published by McGraw-Hill in 1966, has now been reprinted, with correction of misprints, by a new publisher (at 113 Coble Street, Ithaca, New York).